

Constrained Control of an Unstable Biological Reactor

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Investigating stability and multiplicity phenomena when microorganisms are growing in continuous culture is an integral part in the process of understanding biological processes, and in the design, control and optimization of biological reactors. An important application in this field is the production of single-cell protein from methanol. These organisms have been shown to exhibit multiple steady-state behavior.

This paper presents the results of a study of unusual stability behavior exhibited by such a system when feedback control is used to stabilize a naturally unstable steady state and the manipulated variable is constrained. The investigation was carried out to examine the usefulness of a simple growth model previously developed and to further understand the control of unstable processes. Since multiple and unstable steady states in biological systems represent abnormal metabolic conditions our interest is beyond engineering considerations. Demonstration of unstable steady states and other related pathological behavior provides the means by which these otherwise unobservable states can be made accessible. They can then be exploited to provide unique insights into the growth of microorganisms, which might ultimately lead to better predictive models and optimization of specific biological processes.

DiBisio et al. (1981) demonstrated simple feedback control could be used to stabilize unstable steady states in a laboratory biological reactor. Much of the previous work cited there is of relevance to this study. Recently Agar and Bailey (1981) found multiple steady states in a continuous stirred tank biological reactor (CSTBR) and attributed the fundamental cause to be related to an agglomeration phenomenon.

INTRODUCTION

The equations describing the ideal open-loop CSTBR are:

$$\dot{X} = X(\mu - D) \quad (1)$$

$$\dot{S} = D(S_f - S) - \sigma X \quad (2)$$

For this work a simple model describing μ and σ as functions of methanol concentration (S , W/V %) only was used. It was derived from previous work (DiBisio et al., 1981a). Both μ and σ are nonmonotonic one-hump functions of S while Y , ($Y = \mu/\sigma$) is a monotonically decreasing function of S . The qualitative nature of μ and σ has consequences for the stability of the system. Limit cycles are not possible for these equations due to the nature of the yield function. It is easily shown that the unstable steady states are always saddle points while the stable steady states are nodes. The dynamics of these equations were studied by direct numerical integration and subsequent plotting on a phase plane, since global behavior was desired.

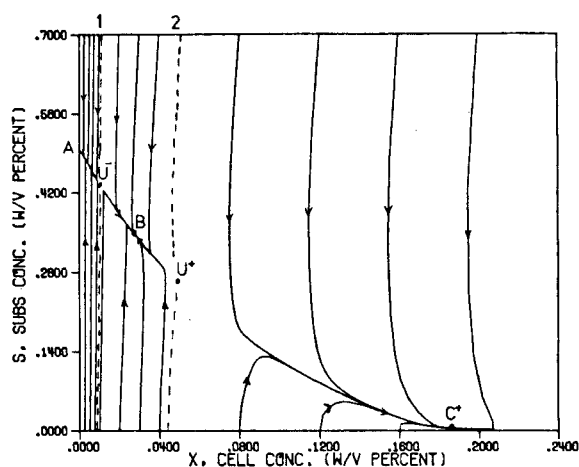
SIMULATION RESULTS

Background

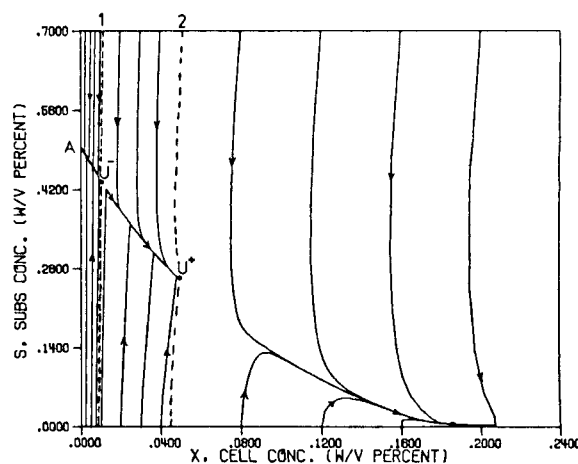
Application of a simple unconstrained P controller has been shown to be sufficient to operate a laboratory reactor at the unstable

steady state. The feedback loop consisted of using cell concentration as the measured variable and feed flow rate (dilution rate) as the manipulated variable. The set point used is the value of the cell concentration at the desired operating point. With sufficiently high proportional gains (DiBisio et al., 1981a,) global stability may be attained.

The presence of predetermined constraints on the manipulated variable can result in the loss of global stability and the possibility of as many as five steady states. A typical example of this is shown in Figure 1a where PI control is utilized, under dilution rate constraints, to attempt to operate the system at point B, the original unstable steady state. The desired operating point B is at a dilution rate of 0.41 h^{-1} , (D_s). One can see the small region of asymptotic



(a)



(b)

Figure 1. (a) Simulated phase plane plot for PI control of bioreactor at an unstable steady state with dilution rate constraints ($0.39 \leq D \leq 0.43 \text{ h}^{-1}$, $K_c = 5 \text{ h}^{-1}/(\text{W/V } \%), \tau_I = 100 \text{ h}$). (b) Simulated phase plane plot for cascade control of bioreactor to the lower dilution rate constraint ($K_{c1} = 5 \text{ h}^{-1}/(\text{W/V } \%), \tau_I = 100 \text{ h}, K_{c2} = 0.01$).

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stability which is a result of the presence of the constraints. When the controller is saturated at the upper dilution rate limit, 0.43 h^{-1} , a new stable steady state (C^+) is possible. The point represented by U^+ corresponds to the unstable steady state of D^+ . In the opposite direction, saturation of the controller at the lower constraint, 0.39 h^{-1} , results in washout to point A. The new unstable steady state given by U^- is that at D^- . These particular conditions and constraints were chosen as a matter of experimental convenience. The five steady-state phase plane will always exist for nonmonotonic growth functions when the upper constraint is less than the maximum growth rate and the lower constraint is nonnegative.

Cascade Control

To preserve the integrity of the inner PI loop with its constraints, and demonstrate the existence of the unstable states at U^- and U^+ , a cascade loop is required. The setpoint of this controller would be the cell concentration at U^- or U^+ and its output would be the setpoint of the inner loop controller. The overall loop would still be subject to the constraints on the dilution rate, and therefore either U^- or U^+ could be approached only from inside the region bounded by the separatraces #1 and #2 in Figure 1a.

The equations for this control system are given below:

Inner loop:

$$D = D_s + K_{c1}(X - X_{sp}) + \frac{K_{c1}}{\tau_I} \int_0^t (X - X_{sp}) dt \quad D^- \leq D \leq D^+ \quad (3)$$

Cascade loop:

$$X_{sp} = X_d + K_{c2}(X - X_d) \quad (4)$$

The setpoint to the cascade controller is given by x_d , and that of the inner loop by X_{sp} . The result obtained when applying cascade control to reach the unstable steady state at the upper constraint (D^+) designated by U^+ is shown in Figure 1b.

The initial conditions within separatraces #1 and #2 can now be made to converge upon point U^+ . Those initial conditions outside of this range result in either washout or C^+ since there is effectively no control in these regions of the phase plane. Thus, U^+ is now metastable. If any perturbations in X or the setpoint (or improper control constants) occur that drive the system past either U^+ to the right or U^- to the left then the stable steady state, C^+ , or washout, respectively, will be obtained. Similar results were obtained for control to U^- . Attempts were then made to demonstrate this behavior experimentally.

EXPERIMENTAL RESULTS

Background

The objectives of this part of the work were to use the knowledge of control at unstable steady states and the results of the phase-plane analysis to investigate this system with a constrained PI controller. Primarily this would involve obtaining the desired unstable steady state using a PI controller with a constrained manipulated variable (the dilution rate), and then using cascade control to demonstrate the existence of the unstable steady states at the constraints.

Methods and Materials

The experimental system was fundamentally the same as that described in DiBiasio et al. (1981a), except that a PDP 11-34 minicomputer was used to implement the control loop.

PI Control with Constraints

A complete run for constrained PI control (without the cascade loop) to the unstable steady state D_s (point B in Figure 2) is shown in Figure 2a.

The stability of the system with this constrained controller was then tested. This was done by introducing a negative cell concentration per-

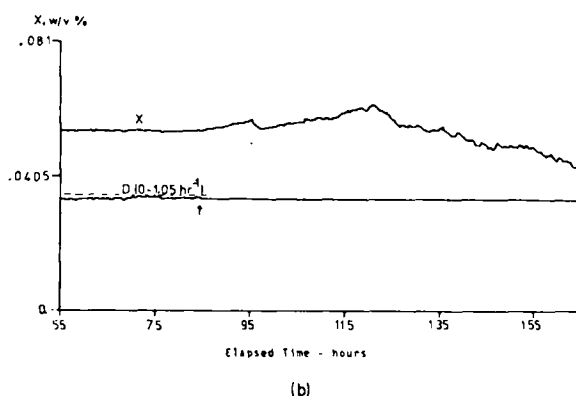
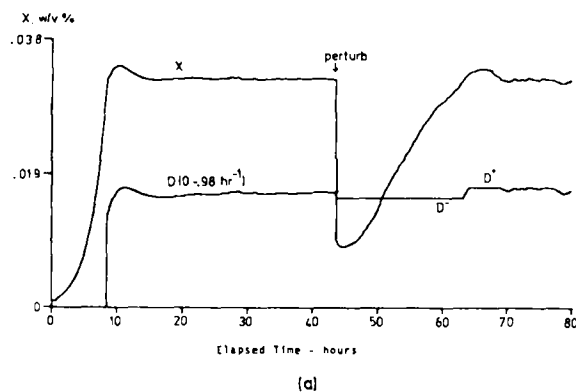


Figure 2. (a) Experimental stability test of bioreactor under PI feedback control with dilution rate constraints. ($D_s = 0.41 \text{ h}^{-1}$, $D^+ = 0.43 \text{ h}^{-1}$, $D^- = 0.39 \text{ h}^{-1}$, $K_{c1} = 125.0 \text{ h}^{-1}/(\text{W/V } \%)$, $\tau_I = 2.8 \text{ h}$). (b) Experimental cascade control of bioreactor to the upper dilution rate constraint. Cessation of control occurs at arrow. ($D^+ = 0.43 \text{ h}^{-1}$, $K_{c1} = 125.0 \text{ h}^{-1}/(\text{W/V } \%)$, $\tau_I = 2.8 \text{ h}$, $K_{c2} = 0.1$).

turbation. A volume of the reactor fluid was removed and quickly replaced with sterile media. This resulted in a decrease in cell concentration as shown by the arrow in Figure 2a. The controller became saturated at D^- . Eventually it returned to the steady state indicating the stability of the control loop. The net result of the perturbation can be better described by referring to Figure 1a. After being at the steady state given by point B, the perturbation resulted in a step change in the reactor conditions to a point just within the region bounded by separatraces #1 and #2. The system should then return to point B as it is asymptotically stable in that region. This was confirmed.

Cascade Control

The possibilities of using cascade control to stabilize the unstable steady states given by U^- and U^+ were then investigated.

The results of one typical run are presented in Figure 2b. Due to the system disturbances such as air bubbles, cell clumps, and possible wall growth in the spectrophotometer flow tube, it was in practice, a physical impossibility to obtain decent control exactly at the constraint. Small disturbances which manifested themselves as relatively large changes in measured optical density would cause saturation of the controller and drive of the cell density away from the desired point. This is a reflection of the metastability of points U^+ and U^- during cascade control. For these reasons there was a small offset between the steady state reached and the actual constraint seen by the reactor.

The results of Figure 2b show that good control was evident at a flow value as close as physically possible to the actual constraint on the pump.

As a test of the stability of point U^+ , after steady state was reached, all control action was removed as indicated by the arrow. The dilution rate was then constant and the instability of the steady state under open-loop conditions was subsequently demonstrated. Similar experiments were conducted which demonstrated cascade control to point U^- .

DISCUSSION

Computer simulations were used to analyze the isothermal stability behavior of a CSTBR using a simple empirical growth model. Phase-plane analysis showed that when the reactor was controlled to a naturally unstable steady state and the manipulated variable was sufficiently constrained, as many as five steady states were possible. Two new unstable steady states appeared corresponding to those at each of the controller constraints. The consequence of this was to change the nature of the control at the desired point from one which is globally stable without constraints, to one which is only locally stable.

Using a digital computer for control and a laboratory continuous flow reactor in which methanol-utilizing organisms were cultured, this behavior was investigated experimentally. Results showed that PI control with constraints was possible and that the closed-loop system was not globally stable. Cascade control was implemented to demonstrate the existence of the unstable steady states at the constraints. The metastable nature of these points was indicated.

The steady state results were in good agreement with predictions from the simple model and were consistent with results expected from a conceptual model of methanol metabolism discussed elsewhere (DiBiasio et al., 1981b). The difference between experimental and simulation controller constants is a reflection of model deficiencies in predicting transient behavior. Other interesting dynamic behavior was observed which was not predicted, and this will be discussed in a later report.

NOTATION

D	= dilution rate (volumetric flow rate/reactor volume)
D^-	$1/h$
D^+	= lower dilution rate constraint, $1/h$
K_{ci}	= upper dilution rate constraint, $1/h$
	= proportional control constant ($i = 1, 2$)

S	= substrate (methanol) concentration, W/V %
S_f	= feed substrate concentration, W/V %
X	= cell concentration, W/V %
X_{sp}	= setpoint to inner control loop, W/V %
X_d	= setpoint to cascade control loop, W/V %

Subscripts

s	= denotes steady state value
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Greek Letters

μ	= specific growth rate, $1/h$
σ	= specific substrate consumption rate, $1/h$
τ_I	= integral control constant, h

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Supercritical Fluid Extraction with Mixed Solvents

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In recent years there has been a revival of interest in supercritical fluid extraction, i.e., in the separation of condensed-phase mixtures by extraction, using a compressed gas (rather than a liquid) as the selective solvent (Schneider et al., 1980). Pertinent phase equilibria have been reported by numerous authors, including, for example, Mackey and Paulaitis (1979), Johnston and Eckert (1981), Kurnik and Reid (1981), and Kurnik et al. (1981). Since the density of a fluid near its critical point is sensitive to small changes in pressure and temperature and since solubility is strongly dependent on the solvent's density, it is sometimes attractive to use a fluid near its critical state as an extraction solvent because solvent regeneration is then easily achieved.

Most phase-equilibrium studies related to supercritical extraction have been restricted to single-solvent systems. However, under certain conditions, it may be advantageous to use a mixed solvent. This paper discusses some of the fundamental considerations leading to such advantages and presents a few examples.

ADVANTAGES OF MIXED SOLVENTS

To minimize operating costs in a continuous extraction process, it is desirable to keep solvent flows as low as possible. Low solvent flows require high solubility of the solute in the gaseous solvent.